MA114 Summer 2018
Worksheet 10 – Comparison Tests
6/26/18

1. Use the appropriate test - Divergence Test, Comparison Test, or Limit Comparison Test - to determine whether the following infinite series are convergent or divergent.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+n^{3/2}}$ $\quad 0 < \dfrac{1}{1+n^{3/2}} < \dfrac{1}{n^{3/2}}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ is a convergent p-series,

so $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+n^{3/2}}$ converges by the Comparison Test.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}} = \sum_{n=1}^{\infty} \frac{n}{n^{5/2}} + \frac{1}{n^{5/2}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} + \frac{1}{n^{5/2}}$ which both are convergent p-series,

so $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$ converges.

c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2+5^n}$ $\quad 0 < \dfrac{2^n}{2+5^n} < \dfrac{2^n}{5^n}$ and $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n$ is a convergent geometric series $\left(r = \frac{2}{5}\right)$

so $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2+5^n}$ converges by the Comparison Test.

d) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^2 - \cos^2(n)}$

Notice that $0 \le \cos^2(n) \le 1$, so $\dfrac{n}{n^2-\cos^2(n)} \ge \dfrac{n}{n^2} = \dfrac{1}{n}$. $(n \ge 1)$

Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (this is the harmonic series), $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^2-\cos^2(n)}$ diverges by the Comparison Test.

e) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n + 2}{3^n + 1}$

$\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{4^n+2}{3^n+1} \overset{L'H}{=} \lim_{n\to\infty} \frac{4^n(\ln 4)}{3^n(\ln 3)} = \infty$ since $\left(\frac{4}{3}\right)^n \to \infty$ as $n \to \infty$.

So $\displaystyle\sum_{n=0}^{\infty} \frac{4^n+2}{3^n+1}$ diverges by the Divergence Test.

f) $\displaystyle\sum_{n=0}^{\infty} \frac{2}{n^2 + 5n + 2}$

$\dfrac{2}{n^2+5n+2} \approx \dfrac{2}{n^2}$, so let's use the LCT:

$$C = \lim_{n\to\infty} \frac{\frac{2}{n^2+5n+2}}{\frac{2}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2+5n+2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1}{1+\frac{5}{n}+\frac{2}{n^2}} = \frac{1}{1+0+0} = 1.$$

So since $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2}$ is a convergent p-series, by the LCT $\displaystyle\sum_{n=0}^{\infty} \frac{2}{n^2+5n+2}$ also converges.